THE "CATTLE PROBLEM." BY ARCHIMEDIES 251 B. C.

By A. H. BELL, Hillsboro, Illinois.

Compute, O stranger! the number of of cattle of Helios, which once grazed on the plains of Sicily, divided according to their color, to wit:

$$\text{1st White Bulls} = \frac{\text{Black Bulls}}{2} + \frac{\text{Black Bulls}}{3} + \text{Yellow Bulls.}$$

2nd Black Bulls=$\frac{1}{4}$ and $\frac{1}{5}$ of the Dappled Bulls+the Yellow.
3rd Dappled Bulls=$\frac{1}{6}$ and $\frac{1}{7}$ of the White Bulls+the Yellow Bulls.
4th The White cows=$\frac{1}{3}$ and $\frac{1}{4}$ of the Black Herd, Bulls and Cows=Herd.
5th The Black cows=$\frac{1}{4}$ and $\frac{1}{5}$ of the Dappled Herd.
6th The Dappled cows=$\frac{1}{5}$ and $\frac{1}{6}$ of the Yellow Herd.
7th The Yellow cows=$\frac{1}{6}$ and $\frac{1}{7}$ of the White Herd.

He who can answer the above is no novice in numbers. Nevertheless he is not yet skilled in wise calculations! but come consider also all the following numerical relations between the Oxen of the Sun.

8th If the White Bulls were combined in one total, with the Black Bulls they would be in a figure equal in depth and breadth and the far stretching plains of Thrinacia would be covered by the figure (square) formed by them.

9th Should the Yellow and Dappled Bulls be collected in one place, they would stand, if they ranged themselves one after another completing the form of an equilateral triangle. If thou discover the solution of this at the same time; if thou grasp it with thy brain; and give correctly all the numbers; O Stranger! go and exult as conqueror; be assured that thou art by all means proved to have abundant of knowledge in this science.—This is translated by T. L. Heath, author of Diophantos, Cambridge, England, 1889.

The first known answer to the Celebrated Cattle Problem by Archimedes 251 B. C. was computed by the Hillsboro, Illinois, Mathematical Club, 1889 to 1893. Edmund Fish, Geo. H. Richards, and A. H. Bell.

The numbers satisfying all of the 9 conditions as given are the very smallest that will meet the requirements and critical tests that are also given. Mathematicians have heretofore obtained the 8th condition which requires the White and Black Bulls to equal a square number, and is 79 450 446 596 004 = □ number; the 9th condition that the Dappled and Yellow Bulls should equal a triangular number is not fullfilled by the corresponding number, 51, 285 802 909 803, which is designated by B. We seek a square multiplier which call x^2 let $Bx^2 = \frac{n(n+1)}{2}$ = the expression for a triangular number which gives $8Bx^2+1=(2n+1)^2=y^2$ and we at once get $\sqrt{8B}=\frac{\sqrt{(y^2-1)}}{x}$. The square root of $8B$ by continued fractions will give x, and then we have

$x^2 =$ 34 555 906 354 559 370 506 303 802 963 617 + 68 829 periods of 3's + 252 058 980 100.

White Bulls 1 596 510 804 671 144 531 435 526 194 370 + 68 834 periods of 3's + 385 150 341 800.

Black Bulls 1 148 971 387 728 289 999 712 359 821 824 + 68 834 periods of 3's + 899 825 178 600

Dappled Bulls 1 133 192 754 438 638 077 119 555 879 202 + 68 834 periods of 3's + 921175 894 000

Yellow Bulls 639 034 648 230 902 865 008 559 676 183 + 68 834 periods of 3's + 635 296 026 300

White Cows 1 109 829 892 373 319 039 723 960 215 824 + 68 834 periods of 3's + 914 059 564 000

Black Cows 753 594 142 054 542 639 814 429 119 589 + 68 834 periods of 3's + 238 562 645 400

Dappled Cows 541 460 894 571 456 678 023 619 942 106 + 68 834 periods of 3's + 608 963 318 000

Yellow Cows 837 676 882 418 524 438 692 221 984 107 + 68 834 periods of 3's + 116 422 113 700

Total 7 760 271 406 486 818 269 530 232 833 209 + 68 834 periods of 3's + 719 455 081 800

W. and B. Bulls 2 745 482 192 399 434 531 147 886 016 194 + 68 834 periods of 3's + 284 975 520 400

Root of above 1 656 949 665 133 506 668 + 34 414 periods of 3's + 357 460 163 020

D. & Y. $= \triangle = 1$ 772 227 402 669 540 942 128 115 555 385 + 68 834 periods of 3's + 556 471 920 300

Root of $8\triangle + 1$ 3 765 344 502 347 205 884 + 34 414 periods of 3's + 363 134 961 201

These enormous numbers using 206545 figures will make numbers one-half mile long. In the computations to this problem difficulties are encountered at every step, wonderfull discoveries in the properties of vast numbers are disclosed at every turn. A new summation of continued fractions with many novel ways used to obtain the exact figures shown can be had of A. H. Bell, Hillsboro, Illinois.